Spacecraft transfer from interplanetary to low near planet orbit by use of aerobraking in Venus’s atmosphere

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In framework of the feasibility studies for the Venus-D project several options for placing a spacecraft in a low orbit near a planet orbit were studied.

The goal of these studies is to determine the optimal variant of spacecraft delivery into a near-Venus orbit. The criterion of optimization is the payload mass.

Several scenarios of transfer from interplanetary orbit onto low Venus orbit are considered beginning from the classical method of applying a rocket engine velocity impulse. As an alternative concept the direct entry into atmosphere is analyzed supposing that after atmospheric drag deceleration the spacecraft leaves the atmosphere, achieving an orbit with apocenter above the atmosphere and pericenter below the planet’s surface. By applying appropriate velocity impulses, the spacecraft is finally transferred into low orbit. The problem of optimal control of aerodynamic forces during this maneuver is solved. It is supposed that the lift force during aerobraking maneuver changes its direction by rotating the spacecraft about its longitudinal axis.

Also intermediate methods to reach low orbit are analyzed when initially the spacecraft is transferred into a high elliptical orbit by engine impulse and by later successive comparatively small aerobraking maneuvers. The maximum overload during these maneuvers is determined by the maximum allowed temperature on the surface of the heat shield of the spacecraft.

Comparison of the described methods is presented taking into account the payload mass, technical risks and overall duration of maneuvers.

Nomenclature

\( c_x \) – aerodynamic drag coefficient;
\( X \) – aerodynamic drag;
\( k_e \) – effective aerodynamic quality;

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θ – angle between planetocentric velocity vector and the local horizon;  
$r$ – distance SC - planet’s center;  
$\mu$ – gravitational constant;  
$m$ – mass of SC;  
$S$ – reference cross section area;  
$\rho$ – atmospheric density;  

I. Introduction

As is well known, transfer from a flyby hyperbolic trajectory to low near planetary orbit needs application to the spacecraft (SC) of a sufficiently large $\Delta V$. So it is worth analyzing alternative approaches such as the use of aerodynamic braking for this purpose. This study finds the maximum gain in characteristic velocity ($\Delta V$) a SC can achieve with a direct entry into the atmosphere with an engine burn following exit from the atmosphere in order to raise the pericenter altitude which occurs after the SC deceleration in the atmosphere.

But implementation of this concept has considerable difficulties caused by the necessity of guaranteeing a narrow enough entry corridor [1, 2]. Besides it is necessary to have a strong enough heat shield to protect the SC from dynamic heating during entry into the atmosphere. It is possible to bypass these difficulties by choosing a compromising approach, when the SC is initially transferred from the hyperbolic approach orbit into a high elliptical one with pericenter having high enough altitude where the influence of the atmosphere is very small. Then by successive small aerodynamic braking near pericenter, the SC is transferred into an almost circular orbit. After decreasing the apocenter to the prescribed altitude an engine burn transfers the SC to an orbit with a pericenter altitude high enough that its aerodynamic deceleration is practically stopped.

To complete this transfer in a minimum time, it is necessary during the whole deceleration process to keep as precise as possible minimal allowed pericenter height, when the aerodynamic heating of SC does not exceed allowed limits.

II. Optimal aerocapture maneuver

In the papers [1, 2] the approach described above is analyzed. Following this concept we analyze the structure of optimal control of SC motion in the atmosphere. As optimal control we consider one requiring the minimum $\Delta V$ at the apocenter whose radius is equal to some prescribed value. It is supposed that the planet (Venus in our case) has a spherical form and it does not rotate (as its atmosphere), and the drag aerodynamic coefficient $c_d$ and aerodynamic quality $k$ (lift to drag ratio) are constant. The control is realized by the change of the roll angle within limits from 0 to 180 degrees. This leads to a change of the projection of the lift vector onto the vertical plane from its maximum value $Y_{\text{max}}$ to its minimum one - $-Y_{\text{max}}$. It is accepted that the sign of the projection on the vertical plane coincides with the sign of the lift projection onto the radius-vector of the SC. Let us introduce the effective aerodynamic quality $k_e$ that allows expressing the projection of lift on the vertical plane as $Y = k_e X$ where $X$ is aerodynamic drag.
Then control of the motion in the vertical plane is possible by considering changing the effective aerodynamic quality between limits \( -k \leq k_e \leq k \). Using the above nomenclature it is possible to write the differential equations of SC motion as follows:

\[
\frac{dV}{dt} = - \frac{c_x S}{2m} \rho V^2 - \frac{\mu}{r^2} \sin \theta, \quad (1)
\]

\[
\frac{dr}{dt} = V \sin \theta, \quad (2)
\]

\[
\frac{d\theta}{dt} = \frac{1}{V} \left[ k_e \frac{c_x S}{2m} \rho V^2 + \left( \frac{V^2}{r} - \frac{\mu}{r^2} \right) \cos \theta \right]. \quad (3)
\]

Let us introduce \( \sigma = \frac{c_x S}{2m} \) and accept a new simplifying assumption as given in [4]. Supposing that the \( \theta \) angle is small we linearize equations (1) – (3) with respect to \( \theta \). We let the second term in equation (1) to be equal zero ignoring its value as compared with the first term. Also we admit that

\[
\frac{V^2}{r} - \frac{\mu}{r^2} = \frac{1}{r_0} - \frac{\mu}{r_0^2}.
\]

Where \( r_0 \) is distance to the SC at the moment of entry of the SC into the atmosphere. Excluding time \( t \) from the system of equations (1) – (3), we obtain

\[
\frac{dr}{dV} = - \frac{\theta}{\sigma \rho V'} \quad (4)
\]

\[
\frac{d\theta}{dV} = \frac{1}{V} \left[ -k_e + \frac{1}{\sigma \rho V^2} \left( \frac{\mu}{r_0^2} - \frac{V^2}{r_0^2} \right) \right]. \quad (5)
\]

Thus we have unautonomous system with two phase coordinates \( r, \theta \), independent variable \( V \) and control variable \( k_e \).

With the given apocenter distance of the orbit, computed after exit from the atmosphere, the minimum \( \Delta V \) impulse that increases the perigee height to the prescribed value, is reached when maximum velocity is achieved after SC exit from the atmosphere. So the problem of optimum control can be considered as the choice of the control
function $k_e = k_e(V)$ on the intercept $-k \leq k_e \leq k$, which transfer phase point from initial position $(r_0, \theta_0)$ with initial velocity $V = V_0$ into position $(r_k, \theta_k)$ in accordance of the requirement that $r_a(r_k, \theta_k, V_k) = r_{ag}$ and providing $V_k = V_{\text{max}}$. For the use of maximum principle it is enough to change variables $W = -V$ and to apply theorem 5 from the book [5]. According to this theorem we will write function $H$ like:

$$H = -\frac{d \theta}{d V} - \frac{\varphi_2}{d V'}$$

(6)

where $\varphi_1$ and $\varphi_2$ is the solution of a conjugate system of equations (here we again use the variable $V$):

$$\frac{d^2 \varphi^2}{dV^2} + \frac{1}{V} \frac{d \varphi_2}{d V} + \frac{V^2}{\sigma \rho^2} \frac{d \rho}{d r} \frac{\varphi_2}{d \sigma^2 \rho^2 V^2} = 0$$

(7)

If control is optimal, then $H = H_{\text{max}}$ on this control for any $V, V_f \leq V \leq V_0$ and it takes place as it is seen from (4) – (6) for $k_e = k \text{sign} \varphi_2$. The number of the control switches is equal to the number of zeroes of the variable $\varphi_2$ on the interval $V_f < V < V_0$.

$$\frac{d \varphi_2}{d V} = \frac{\varphi_1}{\sigma \rho V'}$$

(8)

The equation (9) by substitution

$$\varphi_2(V) = z(V) \exp \left( -\frac{1}{2} \int \frac{d \xi}{\xi^2} \right)$$

is transformed to the equation

$$\frac{d^2 z}{d V^2} + B(V) z = 0,$$

(10)

where

$$B = \frac{1}{4V^2} + \frac{V^2}{\sigma \rho^2} \frac{d \rho}{d r} \frac{1}{\sigma^2 \rho^2 V^2}.$$

Functions $\varphi_2(V)$ and $z(V)$ are becoming zero simultaneously. According to [6], if the $B(V)$ function for $0 < \varepsilon \leq V < \infty$ satisfies the inequality $B(V) \leq 1/4V^2$ then any nontrivial solution of equation (10) has not more than one zero. Because $d \rho/dr < 0$, then $B(V) \leq 1/4V^2$ if $V^2 \rho_0 - \mu \rho_0^2 \geq 0$. So the optimal control in this problem has not more than one switch, if the velocity on the end of braking is not less than the circular velocity. This conclusion coincides with the result of paper [7] where the atmosphere is supposed to be isothermic.

If at the end of aerobraking the condition $r_a = r_{ag}$ will be cancelled, then from the transversal condition one receives at the end of the trajectory $\varphi_2 = 0$. According to [5] in that point the inequality $H \geq 0$ is to be satisfied. Because at the end of braking we always have $\theta > 0$ then from the inequality $H \geq 0$ for the final point we receive $\varphi_1 \geq 0$. Thus function $\varphi_2$ has only one zero at the end of trajectory (for $V = V_f < V_0$) and it is positive on the interval $(V_f < V \leq V_0)$. So the optimal control function can here be only the positive constant $k_e = k$. This result is understandable enough.
The effectiveness of the optimal control has been estimated by calculations that allowed determining the width of the entry corridor in terms of the pericenter heights. The width of the corridor was determined as a function of the lift-drag ratio $k$ for two values of hyperbolic entry velocity: 2.5 km/s and 6.5 km/s. The lower limit for pericenter heights was determined by the maximum allowed characteristic velocity equal to 0.2 km/s and the upper limit by the apocenter distance equal to 87580 km that corresponds to an orbit with a period of 105 days. The option with a constant value of the lift/drag ratio equal to its maximum value was the optimal control function. The results of this modeling are shown in Fig. 2. The atmosphere model corresponds to the maximum density case [8]. Solid lines are plotted for constant $k$ and dashed ones show the optimal control. As one can see in Fig. 2 optimal control allows an entry corridor width of 8 km for $k=0.5$, $V_\infty = 6.5$ km/s, and for $k=1$, a width of 64 km. With increasing $k$, the effectiveness of optimal control is increasing. For example for $k =1$ the corridor becomes larger by 78% as compared with the case with constant lift-drag. It should be mentioned that, considering contemporary technology, one could expect that the value of $k$ does not exceed 0.5. So the problem of the corridor width can’t be ignored.

### III. Successive aerobraking

Here there are some possibilities to realize some compromising method of SC atmospheric braking. It is supposed that the SC is equipped with an accelerometer that gives measurements used for calculation of the required engine $\Delta V$s to be applied near the apocenter in order to sustain the prescribed deceleration regime.

The goal of this analysis is estimation of the required duration of braking when keeping a maximum balanced temperature of the SC surface and determination of acceptable errors in the knowledge of the SC orbital parameters, and also estimation of the gain in required characteristic velocity ($\Delta V$) (as compared with direct transfer into a low almost circular orbit using only the rocket engine).
The problem is being solved for the central gravity field which is for our goals, quite satisfactory. Also it is supposed that the surface of the planet (Venus for our case) and its atmosphere are spherical and do not rotate; and aerodynamic lift of the SC is equal to zero, with the ballistic coefficient constant.

It is known that the heat flow, generated by aerodynamic braking in case of free molecular flow approximately may be written by the formula from Ref. [3]:

\[ q_g = \alpha \frac{\rho V^2}{2} \sin \theta, \]  

where \( \rho \) is density of the incident flow, \( V \) is velocity of the SC, \( \alpha \) is an accommodation coefficient (0< \( \alpha \)<1), \( \theta \) is the angle between the velocity vector of the SC and element of the SC surface. As one can see from formula (1) if \( \alpha = 1 \), then the value of \( q_g \) is the power of the incident flow per surface unit of the forward part of the SC in the incident gas flow, then supposing \( \alpha = 1 \) it is possible to estimate the upper limit of aerodynamic heat flow for any regime of the stream.

The maximum balanced temperature is determined by the maximum value of the heat flow to SC

\[ q = q_r + q_g, \]  

where \( q_r \) is the heat flow radiated by the Sun and by the planet. With good enough accuracy \( q_r \) can be considered as constant near the planet’s orbit. Then the maximum total heat flow takes place when \( q_r = q_{max} \).

Taking into account that the overload is

\[ n = \frac{\rho V^2 S c_x}{2 mg}, \]  

\( S, c_x, m \) are the area of cross section, aerodynamic drag coefficient and SC mass, \( g \) is acceleration of free motion near Earth surface accordingly) we rewrite formula (11) as (\( \alpha = 1, \theta = \pi/2 \))

\[ q_r = n V \frac{mg}{S c_x}. \]  

Hence the maximum balanced temperature is reached when \( n V = (nV)_{max} \). In case of motion with zero lift with parameters guaranteed to exit from the atmosphere, it is possible to suppose that \( n V = (nV)_{max} \) in the pericenter of the orbit, i.e.
\[(n\pi V)_\text{max} = n_{\pi} V_{\pi}. \tag{15}\]

It is obvious that the duration of apocenter decreasing to the given altitude will be a minimum if on each orbit the maximum balanced temperature will be equal to the maximum permissible limit \(T_p\). As it follows from (15) this is equivalent to the condition

\[n_{\pi} V_{\pi} = (n_{\pi} V_{\pi})_p, \tag{16}\]

where \((n_{\pi} V_{\pi})_p\) is determined by the value of \(T_p\) according to the formula easily found from the well-known equation for balanced temperature, and (12) and (14):

\[(n_{\pi} V_{\pi})_p = (\varepsilon \sigma T^4 - q_p) \frac{5c_v}{mg}. \tag{17}\]

(\(\varepsilon\) is the coefficient of blackness, which is set to one and \(\sigma\) is the Stephan–Boltzmann constant).

If orbital parameters after some time since entry into the atmosphere are known and the control impulse in apocenter \(\Delta V_\alpha\) is to be determined as required for satisfying equation (16) during the next passage through the atmosphere, then equation (16) is possible to consider as an equation relative to \(\Delta V_\alpha\). To solve it the density of atmosphere is to be given as a function of altitude. For known parameters of motion at the instant of beginning of the atmospheric part of the preceding orbit, the last requirement is not necessary because the measurements of overload allow calculation of the density after integration of motion of the differential equation in the atmosphere. The accuracy of the density determination and hence as a result the accuracy of \(\Delta V_\alpha\) impulse accuracy depends on the preciseness of knowledge of the motion parameters and of measurements of the overload.

During calculations done with the use of numerical integration of the system of differential equations, control of the SC is simulated. Influence of errors in knowledge of the radius of pericenter on the accuracy of the overload value follows from equation (16). During this procedure the value of control impulses near apocenter was calculated for Kepler motion as a function required change of radius of virtual pericenter determined as a result of solving equation (16). The equation was solved by a method of successive approaches to the solution with the use of the formula

\[\Delta n_{\pi} = \frac{1}{\beta} \ln \frac{n_{\pi f}}{n_p}, \tag{18}\]

where \(n_{sf}\) is the forecasted value of the overload at pericenter in case of motion without control on the current orbit and \(n_p = (n_{\pi} V_{\pi})_p / V_{\pi}\). The inverse scale of heights \(\beta\), included in formula (18), was calculated with the use of simulated measurements of overload near pericenter for orbital parameters that are different from the real parameters according to the expected values of error in knowledge of pericenter height. For eccentricity exceeding 0.2 as confirmed by the calculations it is possible to use formula (18) without iterations and \(n_{sf}\) can be determined by linear extrapolation of the overload values \(n_x\) measured on the two previous values.
The results of the calculations are presented in Fig. 4 for required coefficient \( \frac{m}{Sc_x} \) depending on the required duration of transfer from the initial high elliptical orbit to the almost circular one for two different permissible values of balanced temperature of the SC surface. It was supposed that the initial period of the near-Venus orbit is one Earth day. The pericenter initial height corresponds to the overload in pericenter equal \( n = 10^{-5} \). It was supposed that aerobraking is finished when the height of the osculating orbit has reached 600 km value. The model of Venus’s atmosphere used for the calculations was developed by the Keldysh Institute of Applied Mathematics of the Russian Academy of Science.

Motion regimes were analyzed with total duration of aerobraking duration exceeding 10 days. For these regimes the dependence required the \( \frac{m}{Sc_x} \) coefficient for the braking duration to be linear. The total required characteristic velocity consumption (\( \Delta V \)) for braking control was confirmed to be inside the 20 m/s limit. The required control impulses were calculated assuming that the initial pericenter height was the one corresponding to the maximum overload equal \( n = 10^{-5} \). It is obvious that for higher pericenter altitude, than the value taken for our case, the consumption of characteristic velocity will increase. It is explained by the increasing change of the pericenter altitude needed to reach the overload value determined by equation (16). The total braking duration will also increase. Simulation has shown that reaching the chosen value of the overload takes 10 - 20 orbits. If the error in the altitude of pericenter knowledge is \( \pm 100 \) km, then it leads to the error in overload value not exceeding 2% of the nominal value.

Fig.5 gives as a function of the number of orbits \( N \) the following parameters: time of pericenter crossing \( t_\pi \), \( \Delta V_\pi \), \( \Delta V_\alpha \) in apocenter, the required \( n_{\pi g} \) and fulfilled overload \( n_a \). Time \( t_\pi \) is counted from the apocenter on the first orbit. Beginning of each orbit is at apocenter. The ballistic coefficient \( S_{cx}/m=0.1m^2/kg \), the parameter \( (n_\pi V_\pi)_g=45 \) m/s (what corresponds the balanced temperature 390°C), initial orbit period is 1 day, the error in the knowledge of the pericenter radius is 50 km. As one can see from Fig. 5 the whole process to reach the required overload value takes the first 10 orbits. During this time interval the control \( \Delta V \) decreases from 2.8 m/s to practically zero, lowering the pericenter height from 266 km to 158 km. During the following 150 orbits the pericenter altitude decreases only by 7 km and practically to zero; only on the last orbits they begin to raise, reaching 0.14 m/s. As for the overload, it is kept practically at the required value. The whole process of braking lasts 351 orbits and takes 102.5 days.
For direct aerobraking one needs to take into account the necessity to have a large-enough heat shield. The other problem is to satisfy a requirement to maintain a precise enough value of the pericenter height. To ease this requirement, the optimal control of SC motion in the atmosphere is to be applied.

Thus transfer of the SC into a low near-Venus orbit with the use of successive brakings in the atmosphere is possible by applying a very simple algorithm of control and under comparatively weak requirements for the accuracy of the determination of the trajectory parameters. With fixed constant balanced temperature of the SC surface, the duration of aero braking rises linearly with increasing of the $\frac{m}{Sc_x}$ coefficient. Due to low expenditures of $\Delta V$ for transfer operations control, the decrease in the required total $\Delta V$ is excellent as compared with a direct transfer into a low planetary orbit using the SC rocket engine only. Determining the gain for initial orbit tangent to the Venus atmosphere and having eccentricity equal 0.8 we calculate that the savings are 2.2 km/s. It is obvious that such technology leads to increasing time needed to reach the final orbit and may require installation of some light heat shield to keep the duration of the transfer within restricted limits. But for greatly increasing the active life time of contemporary SC, the method described here of reaching a low near-Venus orbit seems to be interesting for further development.

V. References

8. A. Oppeneym, Mechanics , No. 5, 52, 1953.

Figure 5. Transfer orbit parameters dependence as functions of the number of orbits.